The Basel Problem:

The quest to compute $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Mr Leonessi

Inspired by a conversation with Jason Yip
at the Annual Meeting of the British Society for the History of Mathematics

King's College London Maths School, March 2026

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In the meanwhile, the Basel Problem is discussed by:

John Wallis (1616–1703)
Gottfried Wilhelm Leibniz (1646–1716)
Jacob Bernoulli (1654–1705)
James Stirling (1692–1770)

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1997 McKinzie & Tuckey complete Euler's proof, without modern tools, in *Hidden Lemmas in Euler's Summation of the Reciprocals of the Squares*.

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Trigonometric single integrals or identities: (Yaglom and Yaglom 1953 [101]), (Matsuoka 1961 [64]), (Stark 1969 [92]), (Holme 1970 [45]), (Stark 1970 [93]), (Skau and Selmer 1971 [89]), (Giesy 1972 [36]), (Papadimitriou 1973 [73]), (Stark 1978 [95]), (Stark 1979 [96]), (Ransford 1982 [77]), (Russell 1991 [82]), (Kortram 1996 [56]), (Hofbauer 2002 [44]), (Woodhouse 2007 [100]), (Passare 2008 [74]), (Levrie 2011 [59]), (Benko 2012 [7]), (Daners 2012 [24]), (Muzaffar 2013 [69]), (Brink 2014 [13]), (Glebov 2015c [39]), (Lord 2016 [60]), (Moreno 2016 [66]), (Velleman 2016 [98]), (Pąk and Korniłowicz 2017 [72]), (Siklos 2018 [87]), (Ribeiro 2019 [79]), and (Del Vigna 2023 [25]).

Maclaurin/power/elementary series: (Knopp and Schur 1918 [54]), (Choe 1987 [21]), (Kimble 1987 [52]), (Shea 1988/9 [85]), (Dumont 1992 [26]), (Kalman and McKinzie 2012 [51]), (Benko and Molokach 2013 [8]), (Patyi 2013 [75]), (Krause 2014 [57]), (Vermeeren 2018 [99]), (Silagadze 2019 [88]), (Komornik 2022 [55]), (Campbell 2024a [15]), (Campbell 2024b [16]), (Campbell and Levrie 2024b [18]), (Abreu 2025a [1]), and (Abreu 2025b [2]).

Double integrals: (Goldscheider 1913 [40]), (Apostol 1983 [4]), (Lord 2002 [61]), (Harper 2003 [41]), (Ivan 2008 [47]), (Jameson 2013 [49]), (Jameson and Lord 2013 [50]), (Ritelli 2013 [80]), (Glebov 2015a [37]), (Shiu 2016 [86]), (Novac 2017 [70]), (Pause 2018 [76]), (Murty 2019 [68]), and (Markov 2022 [62]).

Hypergeometric series: (Choi and Rathie 1997 [22]), (Choi, Rathie and Srivastava 1999 [23]), (Campbell 2022 [14]), (Rathie and Lim 2025 [78]), and (Levrie 2026 [58]).

Probability: (Fujita 2008 [35]), (Pace 2011 [71]), (Holst 2013 [46]), and (Aste 2024 [5]).

Complex analysis: (Marshall 2010 [63]), (Glebov 2015b [38]), and (Jain 2024 [48]).

Topology: (Rosenberg 1984 [81]).

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Mr Judge's favourite proof of the Basel Problem is geometric, by Johan Wästlund in **2010**. 3Blue1Brown made a video: link in the worksheet.

After the Basel Problem: a Millennium Problem