Appendix: Sources consulted in designing the Basel worksheet

For an overview of the modern history of the Basel problem, and the list of modern solutions, I directly referenced Campbell & Levrie (2026).

Key in understanding Euler's language was the explanation provided by McKinzie & Tuckey (1997).

Dunham (1987) highlights the modernity of the proof by induction of $\sum_{k=1}^{\infty} \frac{1}{k^2+\mathrm{k}} = 1$, which I added as an extension; Dunham explains in detail the original proof by Bernoulli, which I only briefly mentioned in the lesson as "cumbersome."

I found an interesting historical digression on the correspondence between Euler and Goldbach about the upper bounds for the Basel Problem, which I then mentioned in a footnote in the worksheet, in (Ayoub, 1974, p. 1072-1073).

At the end of the worksheet extension, I recommend students to watch 3Blue1Brown's video (2018) presenting a purely geometric proof of the Basel Problem, which was recently proposed in a manuscript by Wästlund (2010). I was first pointed towards this resource by Jeremy Judge, my teacher mentor.

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